Cut-Off Space of Cloverleaf Resonators with Electric and Magnetic Walls

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Abstract—**A planar resonator that has the symmetry of a junction circulator is the cloverleaf one. The purpose of this paper is to describe the isotropic cut-off space of this class of resonator using the finite element approach. Circuits with 3 and 4-fold symmetries and with a magnetic or an electric sidewall are separately dealt with. Standing-wave solutions are included for completeness. The gyromagnetic problem is separately investigated.**

Introduction

An IMPORTANT class of microwave circuits is the planar one [1], [2]. Such circuits include disk, triangular, wye, ring and irregular hexagonal resonators. A number of different mathematical methods such as the Greens Function approach [3], [4], the Spectral Domain approach [5], the Finite Element method [6]–[8], the Transverse Resonance method [9] and the Contour-Integral method [2, 10] have been utilized to analyze these various circuits. A related class of problem is the cut-off space of waveguides with irregular cross sections [8], [11]. One structure that has not been described so far is the cloverleaf one with 3-fold symmetry. This topology may be of value in the design of 3-port junction circulators and other circuits. The schematic diagram of this arrangement is indicated in Fig. 1. The paper includes the description of the isotropic cut-off space and the field patterns of cloverleaf circuits with 3 and 4-fold symmetries using the finite element method. It also includes the splitting of the dominant pair of degenerate modes in the gyromagnetic circuit with 3-fold symmetry and the standing wave solution of a 3-port circulator based on this type of topology. Papers on the finite element method are given in [11]–[18], papers on planar isotropic circuits in [19]–[23], work on gyromagnetic ones in [7]–[9], [24] and calculations on n-port planar circuits in [2], [16]–[18], [25].

It is of note that while the dominant pair of degenerate counter-rotating modes in such a gyromagnetic circuit with magnetic sidewalls coincides with two of the eigen-networks entering in the description of a junction circulator using this type of resonator the dominant in-phase mode coincides with the same circuit using an electric sidewall. Since the first circulation adjustment of this type of circuit relies on a degeneracy between these two different boundary value problems the cut-off space and field patterns of the planar cloverleaf resonator with electric sidewalls is included in this work for completeness.

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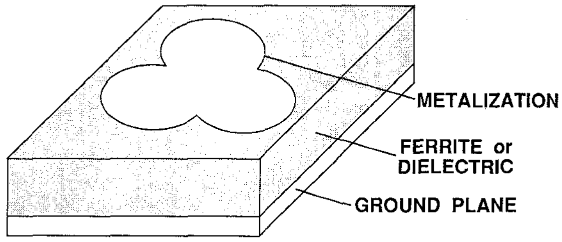

Fig. 1. Topology of cloverleaf planar resonator with 3-fold symmetry.

The Finite Element Method

The cloverleaf circuits investigated in this paper are not compatible with closed form formulations, so that some sort of numerical method is required. A popular approach to this kind of circuit is the finite element one and this is the method employed in this paper.

The mathematical formulation of the finite element method relies on the construction of an energy functional which is then minimized to form a matrix eigenvalue problem. For gyromagnetic circuits with magnetic, electric or mixed electric and magnetic sidewalls one suitable functional is [2]

$$F(E_z) = \iint_s [|\nabla_t E_z|^2 - k_e^2 |E_z|^2]\, ds - j\frac{\kappa}{\mu}\int_\zeta E_z \frac{\partial E_z}{\partial t}\, dt. \quad (1)$$

This functional has the magnetic wall boundary condition included as a natural term. s is the surface of the planar circuit, ζ represents the periphery and t is the boundary tangent defined in a counter-clockwise direction. μ and κ are the diagonal and off-diagonal elements of the tensor permeability. When the parameter κ/μ is set equal to zero the functional describes an isotropic resonator. ∇_t is the transverse differential operator in cartesian coordinates. The wave number (k_e) is defined in the usual way by

$$k_e^2 = \omega^2 \mu_0 \mu_{eff} \epsilon_0 \epsilon_f \quad (2)$$

where

$$\mu_{eff} = \frac{\mu^2 - \kappa^2}{\mu} \tag{3}$$

and ϵ_f is the relative dielectric constant of the ferrite or dielectric substrate.

The finite element method separately involves subdividing the surface of the planar circuit into a number of elementary triangles, m. The number of nodes, n, within each triangle is defined by the degree, q, of the approximation problem as,

$$n = \frac{(q + 1)(q + 2)}{2}. \tag{4}$$

For degree one and two all such nodes lie on the boundary of the triangle, whereas for degree greater than two some nodes lie within the triangle.

The finite element solution continues by approximating the true field solution, E_z, within each triangular finite element by a trial function expansion of the form [4]

$$E_z' = \sum_{k=1}^{n} u_k \alpha_k, \qquad k = 1, 2, \cdots, n \tag{5}$$

where α_k are a suitable set of real basis functions, and u_k are complex coefficients.

The number of nodes, p, within the finite element mesh is not known until the final mesh is assembled since there is no unique relationship between the number of elements and the number of nodes. This is due to the fact that each node is not connected to the same number of triangles. The number of elements, m, for a specified number of nodes, p, is also dependant on the degree of approximation selected. It is therefore possible to have a fine mesh of lower order elements or a coarse mesh of higher order ones. The latter of these possibilities has been shown to be the preferred one [15].

Substitution of the trial function into the energy functional gives the discretized functional as

$$F(\overline{U}) = \overline{U}^{*T}[A]\overline{U} \tag{6}$$

where

$$[A] = \left\{[D] - k_e^2[B] + j\frac{\kappa}{\mu}[C]\right\}. \tag{7}$$

The required eigenvalue problem is now established by minimizing the energy functional using the Rayleigh–Ritz procedure

$$\frac{dF(\overline{U})}{dU_k} = 0.$$

For the case of a gyromagnetic resonator with magnetic sidewalls this becomes,

$$[A]\overline{U} = 0$$

where the matrix of the form $[A]$ is unchanged by the operation. In terms of the original variables the latter equation gives,

$$\left\{[D] + j\frac{\kappa}{\mu}[C]\right\}\overline{U} = k_e^2[B]\overline{U}. \tag{8}$$

The elements appearing in the matrices $[B]$ and $[D]$ are defined in [13] and those in the matrix $[C]$ in [4] as

$$B_{ij} = \iint_s \alpha_i \cdot \alpha_j \, ds \tag{9a}$$

$$C_{ij} = \int_{\zeta} \alpha_i \frac{\partial \alpha_j}{\partial t} \, dt \tag{9b}$$

$$D_{ij} = \iint_s (\nabla_t \alpha_i) \cdot (\nabla_t \alpha_j) \, ds \tag{9c}$$

where,

$$i = 1, 2, 3 \cdots p$$
$$j = 1, 2, 3 \cdots p.$$

Once the basis functions have been selected the general matrix eigenvalue problem may be solved for the p eigenvalues k_e^2 and p eigenvectors $\overline{U}$. The eigenvalues are the normalized cut-off frequencies of the resonator and the eigenvectors are discrete values of the approximated field at the finite element nodes. $\overline{U}$ is a column matrix whose dimension is equal to the number of nodes in the finite element mesh.

Cut-Off Space of Isotropic Cloverleaf Resonator with 3-Fold Symmetry

The cloverleaf circuit discussed in this paper may be described by two radii. Fig. 2 depicts the coordinate system employed here as well as three typical geometries. The minimum outside radius is related to the inside one by

$$R_o(\min) = \frac{\sqrt{3}}{2} R_i \tag{10}$$

The relationship between the details of the circuit and the cut-off wave numbers for the dominant and a number of higher order modes for a resonator with magnetic walls is illustrated in Fig. 3. The finite element mesh utilized in the solution of this circuit is illustrated in Fig. 4. For this arrangement the independent mesh variables are given by

$$q = 2$$
$$m = 57$$

and the dependent one by

$$n = 6.$$

The number of nodes before assembly is

$$m \times n = 342$$

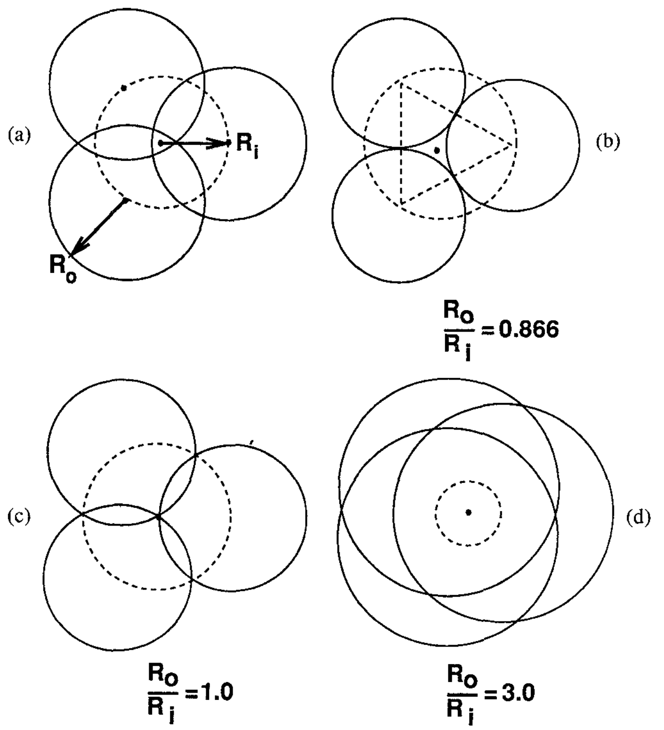

Fig. 2. (a) Co-ordinate system used in description of cloverleaf resonator with 3-fold symmetry. (b) Topology of cloverleaf resonator with minimum surface area $(R_o/R_i) = 0.86$. (c) Topology of cloverleaf resonator with $(R_o/R_i) = 1.0$. (d) Topology of cloverleaf resonator with $(R_o/R_i) = 3.0$.

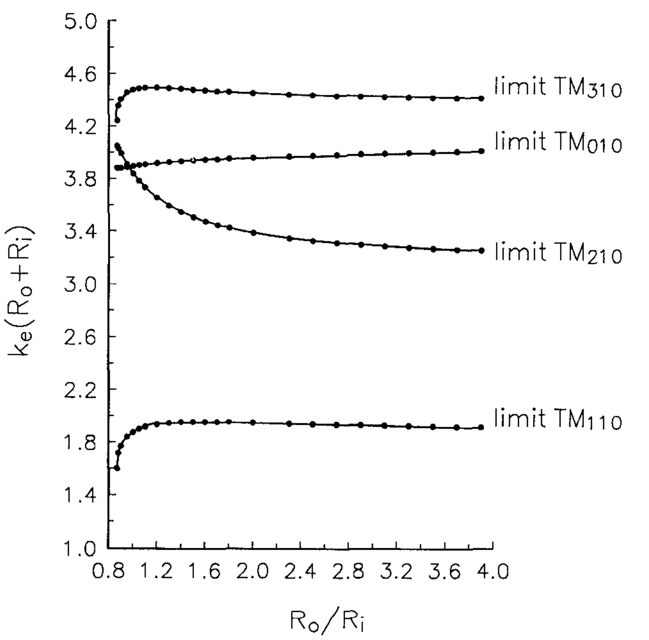

Fig. 3. Cut-off space of cloverleaf planar resonator with 3-fold symmetry with magnetic sidewalls.

and after merging coincident nodes reduces to

$$p = 160.$$

The cut-off numbers of the first four modes in this type of resonator for the condition in (10) are given by

$$k_e(R_o + R_i) = 1.59, \quad R_o = \sqrt{3}\,\frac{R_i}{2}$$

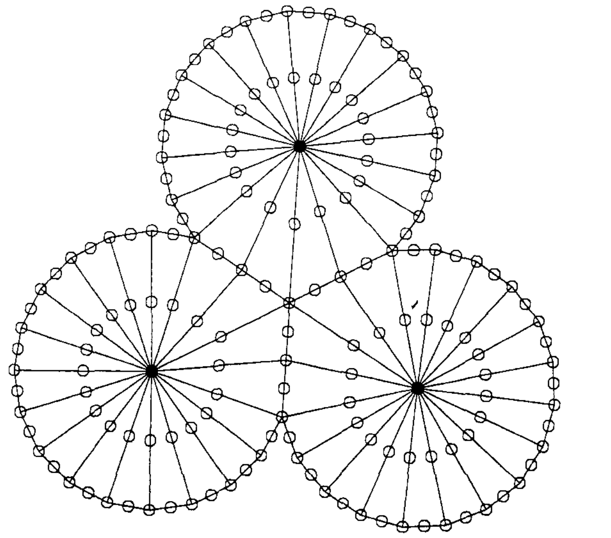

Fig. 4. Finite element mesh for cloverleaf planar resonator with 3-fold symmetry.

$$k_e(R_o + R_i) = 3.88, \quad R_o = \sqrt{3}\,\frac{R_i}{2}$$

$$k_e(R_o + R_i) = 4.05, \quad R_o = \sqrt{3}\,\frac{R_i}{2}$$

$$k_e(R_o + R_i) = 4.24, \quad R_o = \sqrt{3}\,\frac{R_i}{2}$$

One possible notation for this type of circuit may be established by recognizing that as R_o/R_i increases its cut-off space is asymptotic to that of a circular disk resonator. If this convention is adopted then the modes of the cloverleaf resonator may be referred to as limit TM_{mno} modes of the disk resonator. In this nomenclature m refers to the number of half cycles of the magnetic field along the azimuthal direction and n refers to the number of half.cycles along the radius of the circuit, o indicates that there is no variations of the fields along the axis.

One feature of the cloverleaf resonator is the possibility of coupling to it either at the intersection of the lobes employed to define its geometry or at the extremities of them. The former possibility affords some degree at miniaturization in the layout of microwave circuits such as junction circulators. Fig. 5 indicates the relationship between the aspect ratio of the resonator and the ratio of the radii for the two coupling pairs of terminals. The choice of any coupling port is of course related to the external Q factor of the circuit. This problem is however outside the remit of this work.

Cut-Off Space of Isotropic Cloverleaf Resonator with 4-Fold Symmetry

A cloverleaf resonator with 4-fold symmetry with magnetic and electric sidewalls may also be visualized without difficulty. Fig. 6 illustrates its topology. Fig. 7 depicts its construction and three typical geometries. This geometry is defined by variables R_o and A. The minimum

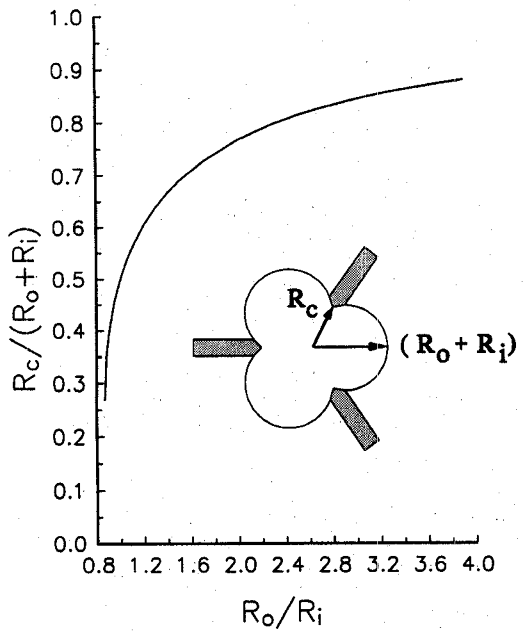

Fig. 5. Relationship between aspect ratio of cloverleaf resonator with 3-fold symmetry and the ratio of the possible coupling port radii.

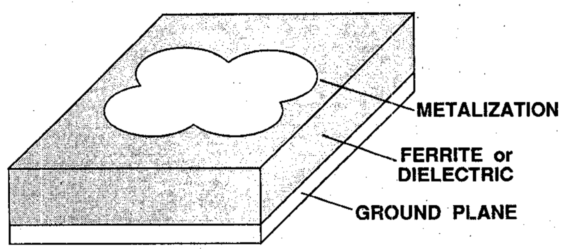

Fig. 6. Topology of cloverleaf planar resonator with 4-fold symmetry.

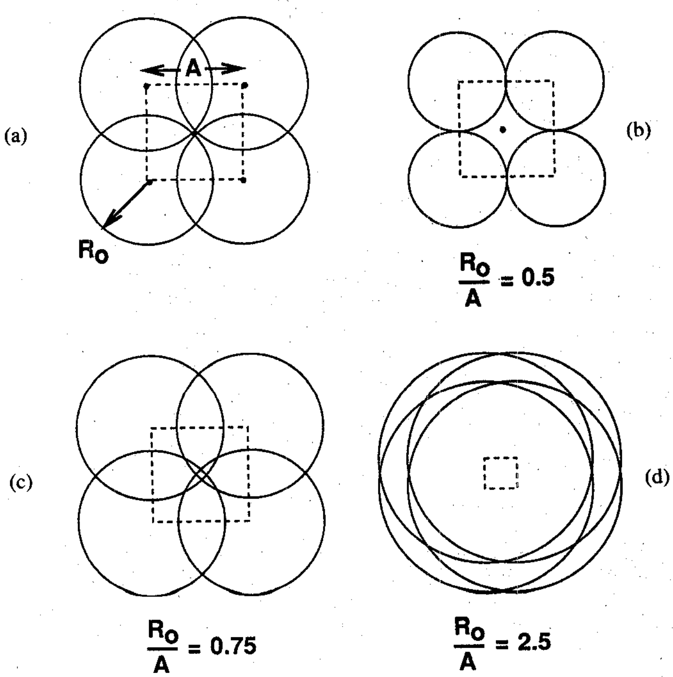

Fig. 7. (a) Co-ordinate system use in description of cloverleaf resonator with 4-fold symmetry. (b) Topology of cloverleaf resonator with minimum surface area $(R_o/A) = 0.5$. (c) Topology of cloverleaf resonator with $(R_o/A) = 0.75$. (d) Topology of cloverleaf resonator with $(R_o/A) = 2.5$.

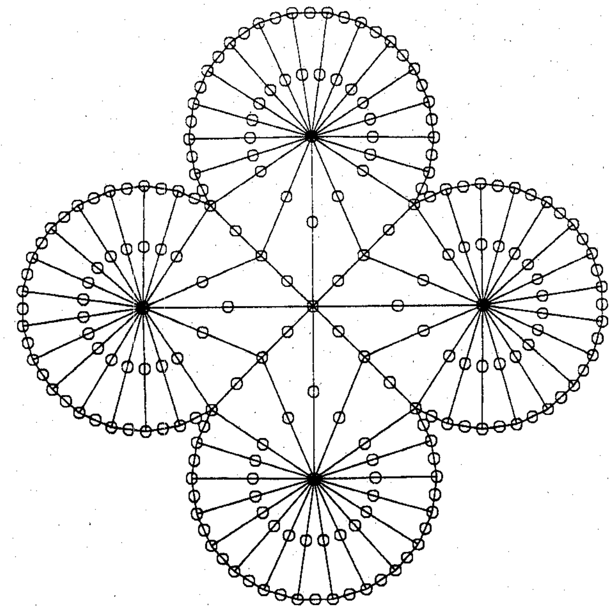

Fig. 8. Finite element mesh for cloverleaf planar resonator with 4-fold symmetry.

outside radius for this circuit is

$$R_o(\text{min}) = \frac{A}{2}. \quad (11)$$

The finite element mesh used in the analysis of this resonator is indicated in Fig. 8. This mesh is described by

$$q = 2$$
$$m = 76$$
$$n = 6$$
$$m \times n = 456$$
$$p = 213$$

The cut-off numbers of this type of circuit bounded by a perfect magnetic wall are indicated in Fig. 9. The mode nomenclature in this instance may also be deduced by referring to a simple disk resonator. A notable feature of this circuit is that as the ratio R_o/A decreases from the disk resonator limit, the degenerate $n = 2$ modes are split by the geometry.

The cut-off numbers of the first 4-modes in this resonator, for the condition in (11) are given by

$$k_e\left(R_o + \frac{A}{\sqrt{2}}\right) = 1.76, \quad R_o = \frac{A}{2}$$

$$k_e\left(R_o + \frac{A}{\sqrt{2}}\right) = 2.18, \quad R_o = \frac{A}{2}$$

$$k_e\left(R_o + \frac{A}{\sqrt{2}}\right) = 4.03, \quad R_o = \frac{A}{2}$$

$$k_e\left(R_o + \frac{A}{\sqrt{2}}\right) = 4.52, \quad R_o = \frac{A}{2}$$

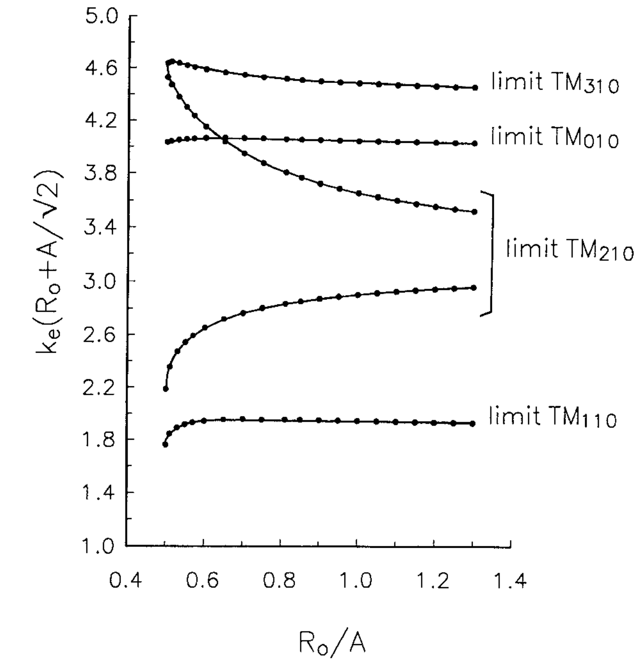

Fig. 9. Cut-off space of cloverleaf planar resonator with 4-fold symmetry with magnetic sidewall.

Field Patterns

The equipotential lines for the first four modes in an isotropic cloverleaf resonator with 3-fold symmetry are indicated in Figs. 10 thru 13 for three different aspect ratios. The symmetric ($n = 0$ limit) mode, illustrated in Fig. 10, has the property that it has a magnetic wall on both the axis and on the open side wall of the planar resonator. The fundamental ($n = 1$ limit) mode is indicated in Fig. 11. Its field pattern, unlike the symmetric mode, displays and electric wall on the axis of the circuit. As the ratio R_o/R_i approaches its lower bound its field pattern reduces to that of the fundamental mode in a symmetric wye resonator with 3-fold symmetry [6]. The first two higher order ($n = 2$ and $n = 3$ limit) modes encountered in this type of resonator are separately indicated in Figs. 12 and 13. The equipotential lines of both these modes also exhibit an electric wall along the axis of the resonator.

The electric field distribution for the first three modes in a cloverleaf resonator with 4-fold symmetry are depicted in Figs. 14–17, again for three typical geometries. The symmetric $n = 0$ limit mode in this sort of circuit once more displays a magnetic wall at the center of the resonator whereas the $n = 1$ and $n = 2$ limit modes have an electric wall there. The dominant mode in this circuit reduces to that of the fundamental mode in a planar X resonator as the ratio R_o/A decreases to its lower bound [6]. Scrutiny of the field patterns for the $n = 2$ limit modes indicates one possible explanation for the splitting at these modes due to the geometry. Fig. 16 shows the lower of the two modes where it is observed that the resonator cut-in in this instance is coincident with an electric wall and will therefore have little effect on the resonant frequency of the mode. For the upper of the two degenerate modes, illustrated in Fig. 17 no such phenomenon exists.

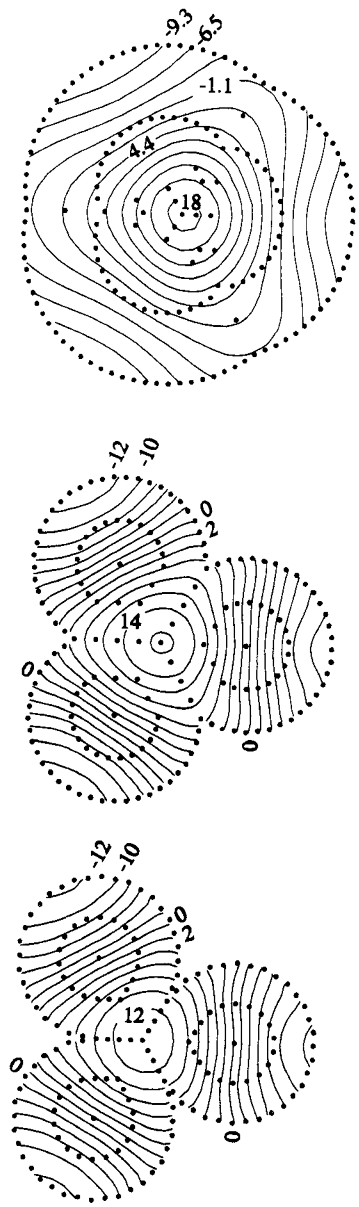

Fig. 10. Equipotential lines of cloverleaf resonator with 3-fold symmetry, for $n = 0$ limit disk mode ($[R_o/R_i]$ = 0.86, 1.0, 5.0).

Split Cut-Off Space of Gyromagnetic Cloverleaf Resonator with 3-Fold Symmetry

The split mode chart of a gyromagnetic cloverleaf resonator is also of some interest. Fig. 18 depicts the relationship between the splitting of the dominant pair of degenerate modes and the aspect ratio of the resonator for six different values of κ/μ. It is obtained by retaining the gyrotropy term in the functional described in (1).

Scrutiny of this result indicates that the gyromagnetic effect in this type of resonator depends upon both the aspect ratio and the gyrotropy and that there exists a number of different combinations of these variables for a given gyromagnetic effect. Another feature of this result is that

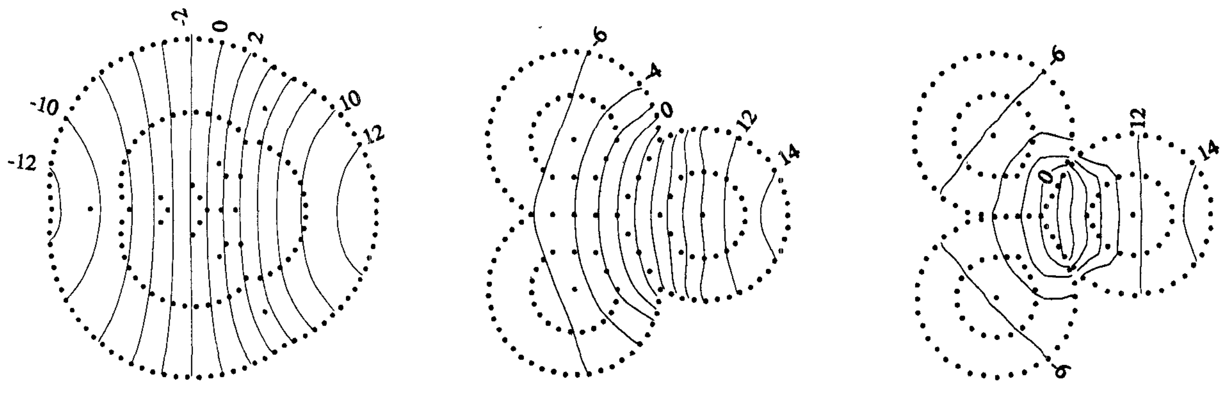

Fig. 11. Equipotential lines of cloverleaf resonator with 3-fold symmetry, for $n = 1$ limit disk mode ($[R_o/R_i] = 0.86, 1.0, 5.0$).

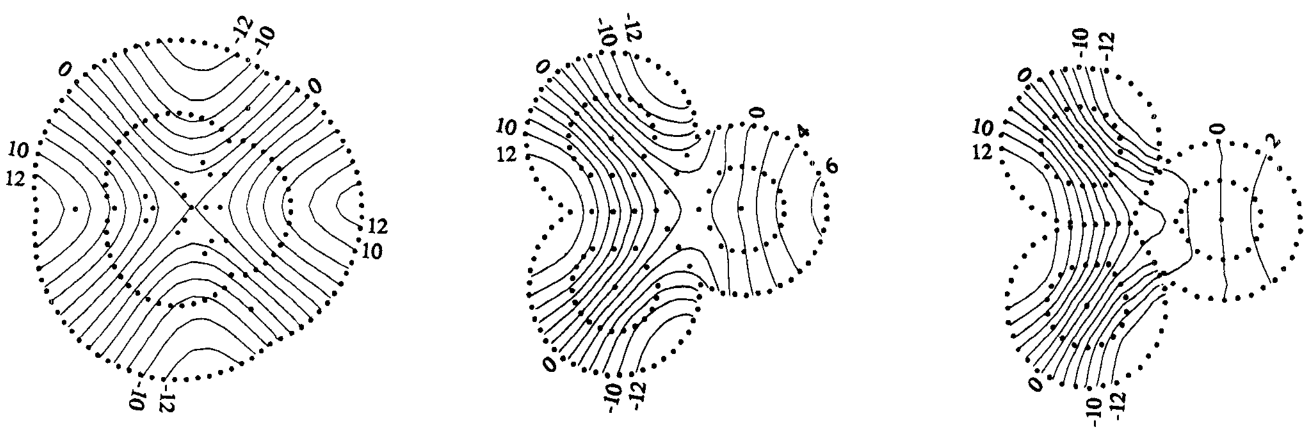

Fig. 12. Equipotential lines of cloverleaf resonator with 3-fold symmetry, for $n = 2$ limit disk mode ($[R_o/R_i] = 0.86, 1.0, 5.0$).

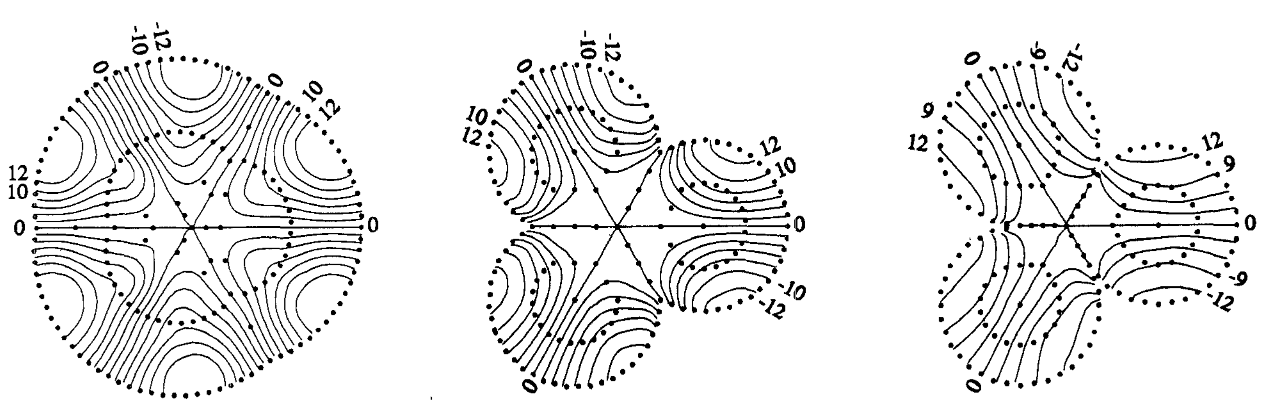

Fig. 13. Equipotential lines of cloverleaf resonator with 3-fold symmetry, for $n = 3$ limit disk mode ($[R_o/R_i] = 0.86, 1.0, 5.0$).

the splitting between the dominant pair of split modes in a weakly magnetized cloverleaf resonator is proportional, in the usual way, to the gyrotropy of the material. It also indicates that the angle between the degenerate split radial wavenumbers of this type of resonator approaches that of a simple disk resonator.

$$\frac{(k_eR)^+ - (k_eR)^-}{(k_eR)} = \frac{2n}{(k_eR)^2 - n^2} \cdot \left(\frac{\kappa}{\mu}\right). \quad (12)$$

It is of separate note that the gyrotropy in this type of resonator is related to the quality factor for a junction circulator using planar resonators by,

$$\frac{1}{Q_L} = \sqrt{3}\left(\frac{\Delta k_eR}{k_eR}\right). \quad (13)$$

Values of Q_L between 2 and $2\frac{1}{2}$ are suitable for the design of quarter-wave coupled devices with modest specifications. This suggests that the normalized split wavenum-

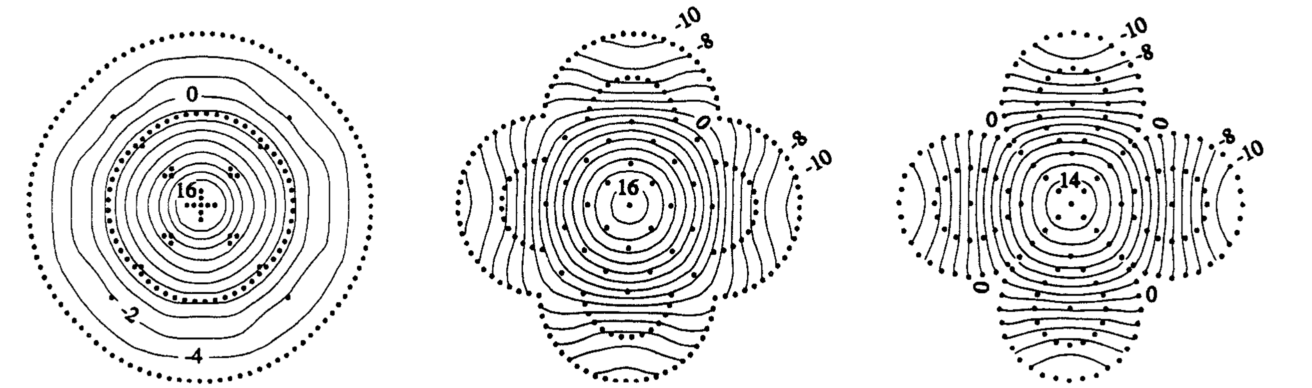

Fig. 14. Equipotential lines of cloverleaf resonator with 4-fold symmetry, for $n = 0$ limit disk mode ($[R_o/R_i] = 0.5, 0.75, 8.0$).

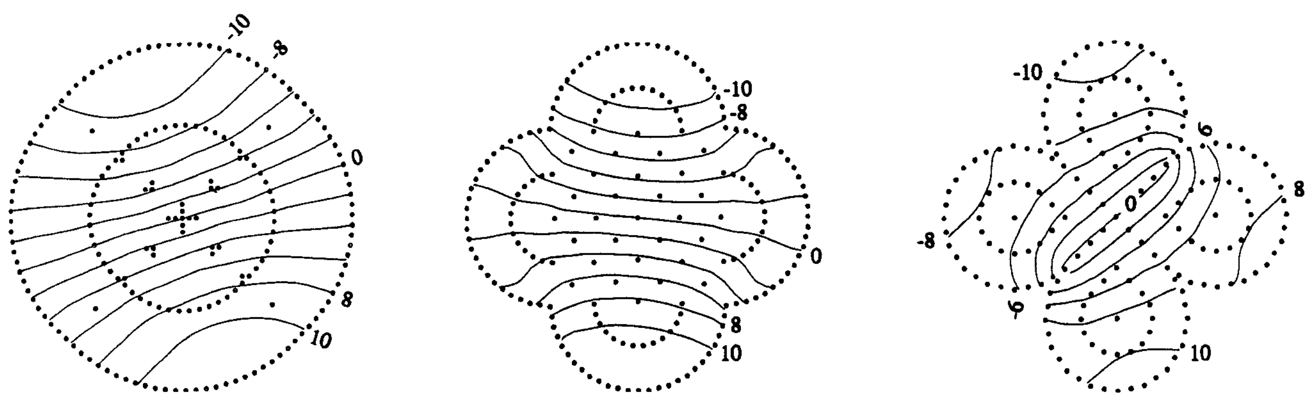

Fig. 15. Equipotential lines of cloverleaf resonator with 4-fold symmetry, for $n = 1$ limit disk mode ($[R_o/R_i] = 0.5, 0.75, 8.0$).

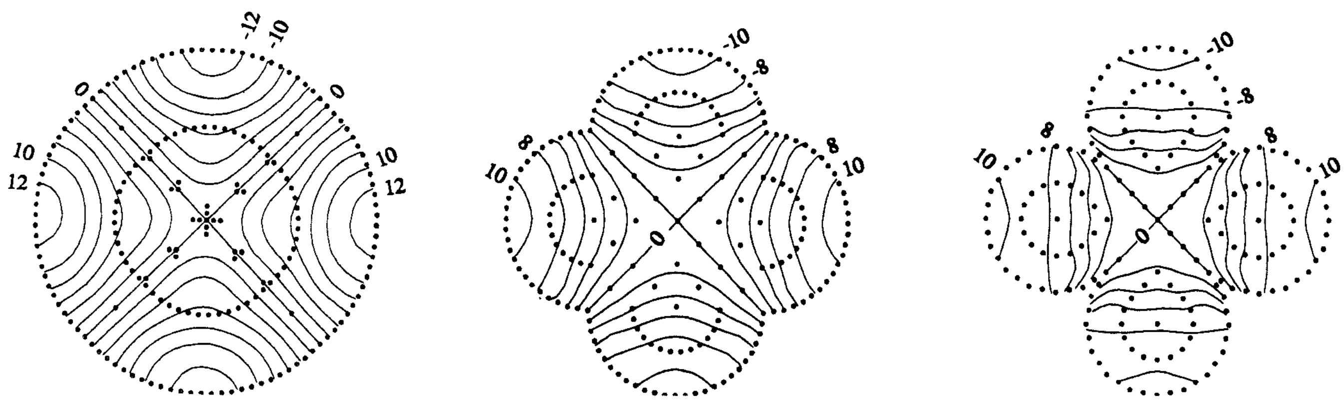

Fig. 16. Equipotential lines of cloverleaf resonator with 4-fold symmetry, for $n = 2$ limit disk (lower) mode ($[R_o/R_i] = 0.5, 0.75, 8.0$).

bers must be bounded by,

$$0.231 < \left(\frac{\Delta k_e R}{k_e R}\right) < 0.288.$$

One attractive feature of the cloverleaf resonator is the possibility of coupling to it at the terminals defined by the intersection of the lobes used in its construction. Adopting a value of $(R_o/R_i) = 1.0$, for instance, indicates that the interval defined by the normalized split cut-off numbers can be accommodated with

$$0.40 \lesssim \frac{\kappa}{\mu} \lesssim 0.60.$$

Once the quality factor of this type of device is established a knowledge of its susceptance slope parameter is sufficient for design. This quantity is defined by the resonator shape, its thickness and the choice of coupling terminals.

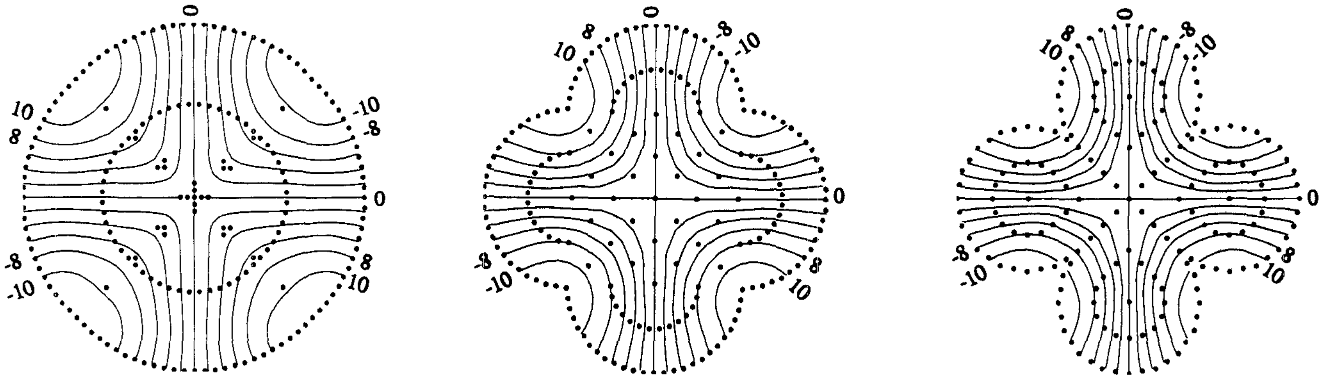

Fig. 17. Equipotential lines of cloverleaf resonator with 4-fold symmetry, for $n = 2$ limit disk (upper) mode ($[R_o/R_i] = 0.5, 0.75, 8.0$).

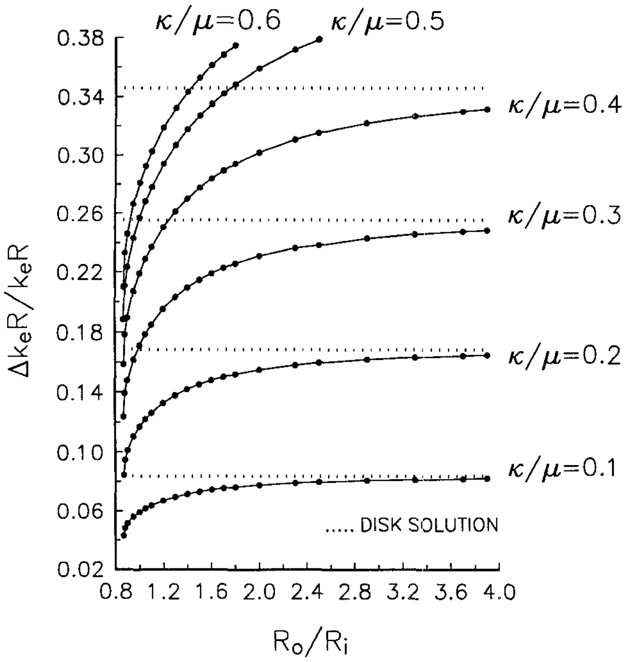

Fig. 18. Split radial wavenumbers of weakly magnetized gyromagnetic cloverleaf resonator.

Standing Wave Solution of Circulators using Cloverleaf Resonators

The standing wave solution of a planar junction circulator using this type of resonator is also of some interest. It may be obtained by taking a suitable linear combination of the modes of the isotropic circuit [6]. Fig. 19 shows the construction of such a solution for the dominant mode in a gyromagnetic cloverleaf resonator with 3-fold symmetry. An ideal circulator may be formed with this type of circuit with ports attached to either two possible triplets of terminals.

Eigenvalues and Eigenvectors of Cloverleaf Resonators with Electric Walls

The finite element approach also lends itself to the description of planar cloverleaf resonators with electric sidewalls. The solution to this problem may be obtained by partitioning the nodal field vectors according to whether the electric field exists or not on the boundary. The discretized functional may then be rewritten as

$$F(\overline{U}) = [\overline{U}_f^* \overline{U}_p^*] \begin{pmatrix} [A_{ff}] & | & [A_{fp}] \\ — & + & — \\ [A_{pf}] & | & [A_{pp}] \end{pmatrix} \begin{pmatrix} \overline{U}_f \\ \overline{U}_p \end{pmatrix} \quad (14)$$

where $\overline{U}_p$ is a column vector containing the forced values of electric field on the electric walls. Since these are zero the problem reduces to that met in connection with (6)

$$F(\overline{U}) = \overline{U}_f^{*T}[A_{ff}]\overline{U}_f \quad (15)$$

where $\overline{U}_f$ is a column matrix containing the unconstrained values of axial electric field and is the unknown of the problem. Applying the Rayleigh-Ritz condition,

$$\frac{\partial F(\overline{U})}{\partial [U_f]_i} = 0$$

reduces the problem once more to the standard form below

$$\left\{[D_{ff}] + j\frac{\kappa}{\mu}[C_{ff}]\right\}\overline{U}_f = k_e^2[B_{ff}]\overline{U}_f. \quad (16)$$

The eigenvalues of this equation, as before, are the cut-off wavenumbers k_e^2 and the eigenvectors are the column matrices $\overline{U}_f$.

The relationship between the geometry of the circuit and the cut-off wave number for an isotropic cloverleaf resonator with three-fold symmetry and an electric sidewall is illustrated in Fig. 20. Once again as R_o/R_i increases its cut-off space is asymptotic to that of the corresponding planar disk resonator.

Field Patterns of Cloverleaf Resonators with Electric Walls

The equipotential lines for a cloverleaf resonator with 3-fold symmetry and bounded by an electric wall are illustrated in Fig. 21 for the first three modes. The dominant mode of this circuit, depicted in Fig. 21(a), exhibits a magnetic wall on the axis and an electric wall at the

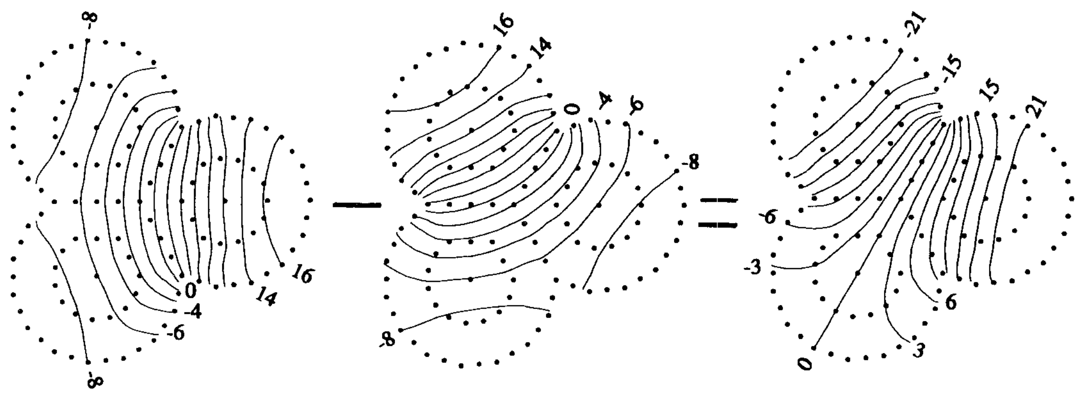

Fig. 19. Standing wave circulation solution of circulator using dominant mode in cloverleaf resonator with 3-fold symmetry ($[R_o/R_i] = 1.0$).

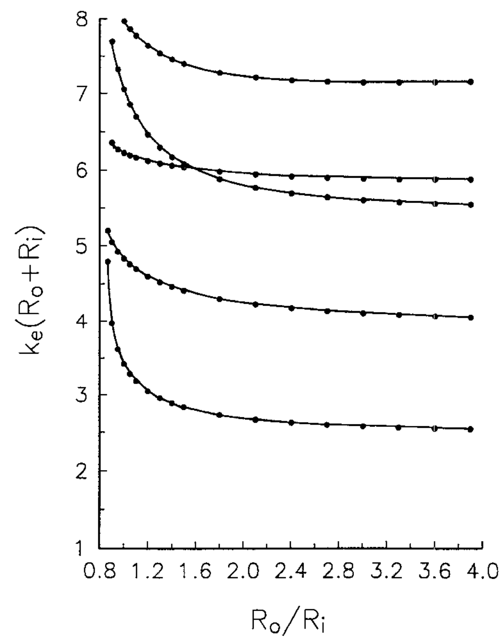

Fig. 20. Cut-off space of cloverleaf planar resonator with 3-fold symmetry with Electric Sidewalls.

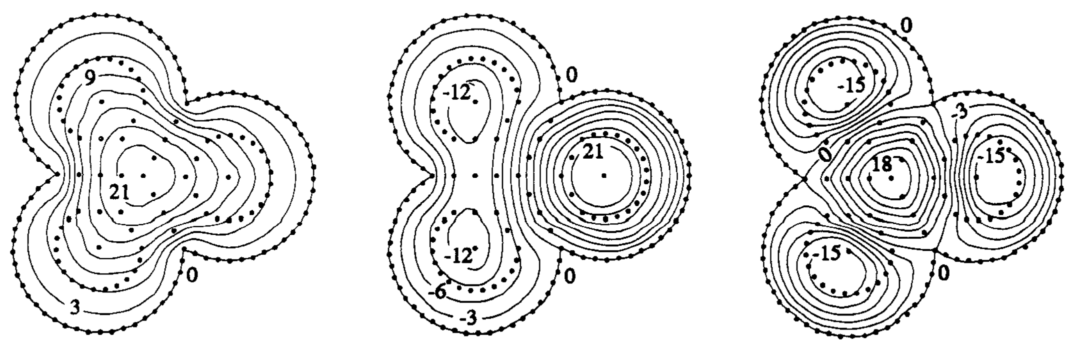

Fig. 21. (a) Equipotential lines of cloverleaf resonator with electric sidewall for dominant mode ($[R_o/R_i] = 1.0$). (b) Equipotential lines of cloverleaf resonator with electric sidewall for first higher order mode ($[R_o/R_i] = 1.0$). (c) Equipotential lines of cloverleaf resonator with electric sidewall for second higher order mode ($[R_o/R_i] = 1.0$).

boundary of the circuit. The first two higher order modes of this type of circuit are indicated in Fig. 21(b) and (c). The first of these has the property that it has an electric wall on both the axis and the sidewall of the resonator. The latter exhibits a magnetic wall at the center and an electric wall approximately midway between the central axis and the perimeter of each of the circles used in the construction of the circuit.

Conclusions

The finite element method has been employed in this paper to investigate the properties of cloverleaf circuits with 3 and 4-fold symmetries and with electric and magnetic walls. The cut-off space of a cloverleaf gyromagnetic circuit with 3-fold symmetry has been separately evaluated. It indicates that there exists a number of different combinations of the aspect ratio of the resonator and its gyrotropy for a given gyromagnetic effect which are in keeping with good engineering practice of this sort of circuit.

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